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ON – LINE IDENTIFICATION OF HYSTERETIC SYSTEM

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Abstract. *An adaptive estimation approach method is used for the on line identification of hysteretic systems under arbitrary dynamic environment. The availability of such an identification approach is indispensable for the on-line control and monitoring of nonlinear structural systems to be actively controlled. The hysteretic restoring force is modeled by the Bouc-Wen model, and the adaptive law insures that all signals will remain bounded. The model is modified to produce a linearly parameterized estimator, which permits the on-line prediction of the hysteretic behavior through recursive techniques.*
The identification of the hereditary nature of the restoring force of this nonlinear system is a significant challenge. However, it is shown through the use of simulation studies and experimental measurements that the proposed approach can yield reliable estimates of the hysteretic restoring force.

Keywords: *Hysteretic Systems, Identification, Adaptive Estimation*

1. INTRODUCTION

1.1. Background

Problems involving the identification of structural systems exhibiting inelastic restoring force with hereditary characteristics are widely encountered in the applied mechanics field. Examples include buildings under strong earthquake excitations or aerospace structures incorporating joints. The restoring force in such systems is hysteretic in nature and therefore cannot be expressed in the form of an algebraic function involving the instantaneous values of the state variables of the system. Consequently, much effort has been directed to develop models of hysteretic restoring forces and techniques to identify these systems. Noteworthy is the work of Baber & Wen (1982), Andronikou & Bakey (1984), Spencer & Bergman (1985), Powell & Chen (1986), Iwan & Cifuentes (1986), Wen & Ang (1987), Worden & Tomlinson (1988), Capecchi (1990), Masri et al. (1991), Loh & Chang (1993), Benedettini et al. (1995), and Chassiakos et al. (1995).

A principal need in actively controlling the nonlinear dynamic response of structural systems undergoing hysteretic deformation is the need for rapid identification of the non-linear restoring force. In this manner, the information can be used by online controlling algorithm to adjust the actuator forces to insure stable response control of the oscillating structure. Recently, (Housner & Masri 1990, 1993) and (Housner et al. 1995) showed that the on-line identification of hysteretic restoring force is indispensable for the practical implementation of structural control.

1.2. Scope

This paper presents a method for on-line identification of hysteretic systems under arbitrary dynamic environments. First, the problem is formulated, and the on-line identification algorithm is next presented. Simulation results of single degree of freedom model is then presented to demonstrate the use and validity of the algorithm under test signals that differ from those used for identification. Additionally, the identification approach is used in experimental measurements in steel subassembly undergoing severe hysteretic deformation. The advantages and limitations of this approach are finally compared to alternative identification methods.

2. DYNAMICS AND MODEL

"Figure 1" shows a single-degree-of-freedom system. The dynamics of the system is described by

$$m\ddot{x}(t) + Q[x(t), \dot{x}(t)] = u(t), \tag{1}$$

where $x(t)$ is the system displacement, $Q[x(t), \dot{x}(t)]$ is the hysteretic restoring force and $u(t)$ is the external excitation. The mass m is assumed to be known, and the measurements of $u(t)$ and $\ddot{x}(t)$ are assumed measurable at time intervals t_s , s = 1, . . . $x(t)$ and $\dot{x}(t)$ is obtained at each time interval by measurement or integration of the acceleration signal.

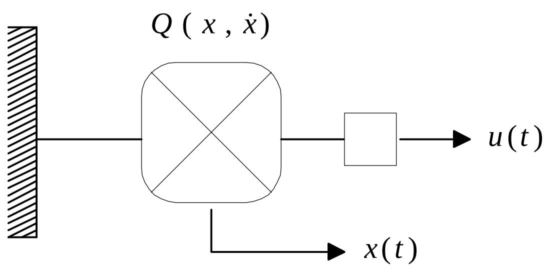

Figure 1- One-degree-of-freedom hysteretic system model

A restoring force with hysteretic characteristic is modeled by a non-linear differential equation. The (Bouc-Wen, 1989) model is chosen to represent this force due to its ability to capture the properties of a wide range of real non-linear hysteretic systems. The equation is given by :

$$Q(x, \dot{x}) = z \tag{2}$$

$$\dot{z} = (1/\eta)[A\dot{x} - \nu(\beta|\dot{x}||z|^{n-1}z - \gamma\dot{x}|z|^{n})] \tag{3}$$

Different combinations of the parameters $\eta, A, \nu, \beta, \gamma$ and n produce smooth hysteresis loops of various hardening or softening characteristics, and different amplitudes and shapes.

At a given instant 's', we have :

$$u(s) = u(t_s) \quad ; \quad x(s) = x(t_s)$$

$$\dot{x}(s) = \dot{x}(t_s) \quad ; \quad \ddot{x}(s) = \ddot{x}(t_s) \quad ; \quad z(s) = z(t_s)\,; \qquad (4)$$

and the system equation of motion, “Eq. (1)”, is rewritten as :

$$Q(s) = z(s) = u(s) - m\,\ddot{x}(s)\,. \qquad (5)$$

“Equation (5)” indicates that the values of z are available at each time interval t_s by using on-line measurements of x, $\dot{x}$, $\ddot{x}$ and u. Therefore, having m, the on-line estimates of the unknown parameters in “Eq. (3)” are determined.

3. ON-LINE IDENTIFICATION

The hysteretic model describing the restoring force,”Eq. (3)”, is given in terms of linearly parameterized coefficients: { $(1/\eta)A$, $(1/\eta)\nu\beta$, $(1/\eta)\nu\gamma$ }, but is non-linear with respect to the power n. Since a linearly parameterized on-line estimator is desirable, the model is modified to include a totally linearly parameterized expression;

$$\dot{z} = (1/\eta)\left[A\dot{x} - \sum_{n=1}^{n=N} a_n \nu\,(\beta\,|\dot{x}||z|^{n-1} - \gamma\,\dot{x}|z|^{n})\right], \qquad (6)$$

where the value of coefficients a_n determines the contribution of power n to the hysteresis, and n is a large enough integer. For example, if power n is 3, and N is chosen as 4, then the coefficients $a_1 = a_2 = a_4 = 0$ and $a_3 = 1$. Now, measurements are taken at discrete time intervals Δt_s, and “Eq. (6)” is rewritten in a discrete-time version to read:

$$\begin{aligned} z(s) = z(s-1) &+ \Delta(t)(1/\eta)A\dot{x}(s-1) + \\ &\Delta(t)\sum_{n=1}^{n=N}[-a_n(1/\eta)\nu\beta\,|\dot{x}(s-1)||\dot{x}(s-1)||z(s-1)|^{n-1}\,z(s-1) \\ &+ a_n(1/\eta)\nu\gamma\,\dot{x}(s-1)|z(s-1)|^{n}]. \end{aligned} \qquad (7)$$

This discrete-time model gives rise to the following discrete-time linearly parameterized estimator :

$$\begin{aligned} \hat{Q}(s) = z(s-1) &+ \theta_0(s)\dot{x}(s-1) + \\ &\sum_{n=1}^{n=N}[\theta_{2n-1}(s)\,|\dot{x}(s-1)|\,|z(s-1)|^{n-1}\,z(s-1) \\ &+ \theta_{2n}(s)\,\dot{x}(s-1)|z(s-1)|^{n}], \end{aligned} \qquad (8)$$

where the coefficients $\theta_i(s)$, i = 0, ... 2N are estimates at time t_s of the corresponding coefficients from “Eq. (7)”, that is $\theta_0(s)$ is an estimate of $\Delta t(1/\eta)A$, $\theta_{2n-1}(s)$ is an estimate of $-(\Delta t)a_n(1/\eta)\nu\beta$, and $\theta_{2n}(s)$ is an estimate of $(\Delta t)a_n(1/\eta)\nu\gamma$.

Now, we define $\theta(s) = [\theta_0(s), \theta_1(s), \theta_2(s), ..., \theta_{2n}(s)]^T$ as the vector the estimates of the system parameters at time t_s, and $\theta^* = [\theta_0^*, \theta_1^*, \theta_2^*, ..., \theta_{2n}^*]^T$ as the vector containing the true value of the parameters. The corresponding system measurements at time t_s are given by the vector :

$$\phi(k-1) = [\dot{x}(k-1), |\dot{x}(k-1)||z(k-1)|^0 z(k-1), \dot{x}(k-1)||z(k-1)|^1, \\ |\dot{x}(k-1)||z(k-1)|^1 z(k-1), \dot{x}(k-1)||z(k-1)|^2, \ldots.., \\ |\dot{x}(k-1)||z(k-1)|^{N-1} z(k-1), \dot{x}(k-1)||z(k-1)|^N]^T. \quad (9)$$

Therefore, the estimator model in (8) may be expressed as :

$$\hat{Q}(k) = z(k-1) + \phi^T(k-1)\theta(k), \quad (10)$$

and the estimation error is :

$$e(s) = \hat{Q}(s) - Q(s) = \phi^T(s-1)\theta(s) - \phi^T(s-1)\theta^* = \phi^T(s-1)\tilde{\theta}(s) , \quad (11)$$

where $\tilde{\theta}(s) = \theta(s) - \theta^*$ is the $((2N+1) \times 1)$ vector of parameter errors between the actual and estimated values $\theta_i(s)$.

From “Eq. (10)” and using the techniques applied in adaptive estimation and control (Ioamnou & Datta, 1991) ; (Polycarpou & Ioamnou, 1992), we adopt the following gradient projection adaptation law :

$$\theta(s) = \begin{cases} \mu(s), & \text{if } \|\mu(s)\| \le M_\theta \\ (M_\theta / \|\mu(s)\|)\mu(s) & \text{if } \|\mu(s)\| > M_\theta , \end{cases} \quad (12)$$

Where:

$$\mu(s) = \theta(s-1) - \frac{\gamma_0}{\beta_0 + \|\phi(s-1)\|^2} e(s-1)\phi(s-1) , \quad (13)$$

and $\gamma_0 > 0$ is the *learning rate* of the algorithm and $\beta_0 > 0$ is a design constant. $\|\phi(s)\|$ and $\|\mu(s)\|$ are the Euclidean vector norms. M_θ is an upper bound number on the norm $\|\theta^*\|$. This upper bound is easily found if some information about the order of magnitude of the elements of θ^* is available.

In “Eq. (12)”, the adaptive law is given as a function of the upper bound of the norm $\|\theta^*\|$ in order to avoid *parameter drift* which may drive the estimate $\theta(s)$ to infinity and lead to an unstable estimate scheme. The equation shows that if $\|\mu(k)\| > M_\theta$ the estimate of $\theta(s)$ remains small in order to maintain the maximum of the norm $\|\theta(k)\|$ equal to M_θ and $\theta(s)$ will not converge to infinity.

The adaptive law of “Eq. (12)” and “Eq. (13)" guarantees that all the signals will remain bounded and the error $e(s) \to 0$ as $s \to \infty$, if the model in “Eq. (3)” is a good representation of the unknown system.

Since the hysteretic model in “Eq. (6)” is a linearly parameterized estimator, recursive on-line estimation techniques can be used and they provide fast identification and output tracking. Other non-linear estimation techniques are disadvantageous since they either search over the whole parameter space spanning a long period (Masri et al. 1980) which renders the technique enviable as an on-line estimation techniques, or increase computational complexity as in the technique based on the steepest descent (Goodwin & Sin 1984).

The representation of the hysteretic model by “Eq. (6)" increases the number of estimated parameters as compared to the original model in “Eq. (3)”. However, the unknown power n is relatively small (typically an integer less than 4), and the total number of parameters is as well relatively small. The system in “Eq. (3)”, linearly parameterized is said to be *system identifiable* under “Eq. (6)”, and the algorithm of “Eq. (12)” and “Eq. (13)” in the sense that the set of parameter vectors θ that satisfy “Eq. (10)” is nonempty. That is, the on-line algorithm should converge to this parameter set. Because of over-parameterization, this parameter set contains more than one point, and the convergence to a *unique* parameter vector cannot be guaranteed. However, tracking of the output signal may be performed even with an over-parameterized model.

4. APPLICATIONS

Identification results from simulated one degree of freedom system and real data of a hysteretic structure are presented to illustrate the applicability of the proposed on-line identification algorithm. After the identification procedure is completed, the parameter values are frozen at their final values. The identified model is validated and tested under various levels of wide-band random excitation.

The identification scheme is next used to track the output of a system which is phenomenologically different from the Bouc-Wen model, and it is seen to produce very good agreement between the predicted and system outputs. Finally, actual testing data acquired from a full-scale structural steel subassembly are used. The identification procedure is shown to identify the hysteretic characteristics and perform very good output prediction.

4.1. Model characteristics

The identification procedure will refer to the hysteretic system of “Fig. (1)”. The following values were given to the model parameters of “Eq. (3)” :

$$\eta = 1, \quad A = 1, \quad \nu = 1, \quad \beta = 1, \quad \gamma = 1, \quad n = 1, \qquad (14)$$

additionally, the system mass was taken as unity.

4.2. Experimental measurements

The experimental measurements for the external excitation $u(t)$ and acceleration $\ddot{x}(t)$ are assumed to be available over a period much longer than the characteristic period of interest. The corresponding system displacement $x(t)$ and velocity $\dot{x}(t)$ are determined by direct measurement or integration of the acceleration signal. The measurements are discretized at time increments $\Delta(t)$ (in this case 0,1 second), and the values of $Q(s)$ are obtained from “Eq. (5)”.

Additionally, the method imposes no restriction on the nature of the excitation source signal. In the case under study, the excitation used is the swept sign signal :

$$u(t) = sin\,((0{,}03t \; + \; 0{,}2)\; t)\,. \qquad (15)$$

The time-history records of the swept-time input excitation and response displacement corresponding to the single degree of freedom excitation system is shown in “Fig. (2)”.

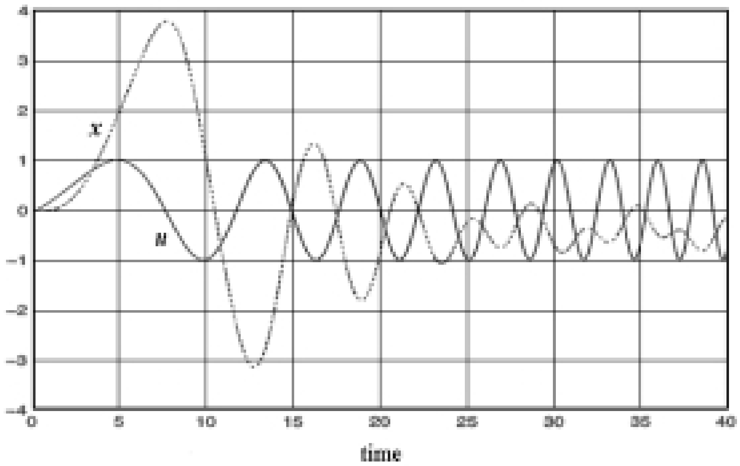

Figure 2- Time-history records of the system excitation and response to a sinusoidal input

4.3. Identification of the hysteretic restoring force

The unknown parameter vector θ is initially set to zero. A value of $n = 3$ and an upper bound $M_{\theta} = 1$ are chosen. According to "Eq. (8)", the algorithm will estimate a total of seven parameters. The on-line estimate of parameters θ_0 to θ_6 is shown in "Fig. (3)".

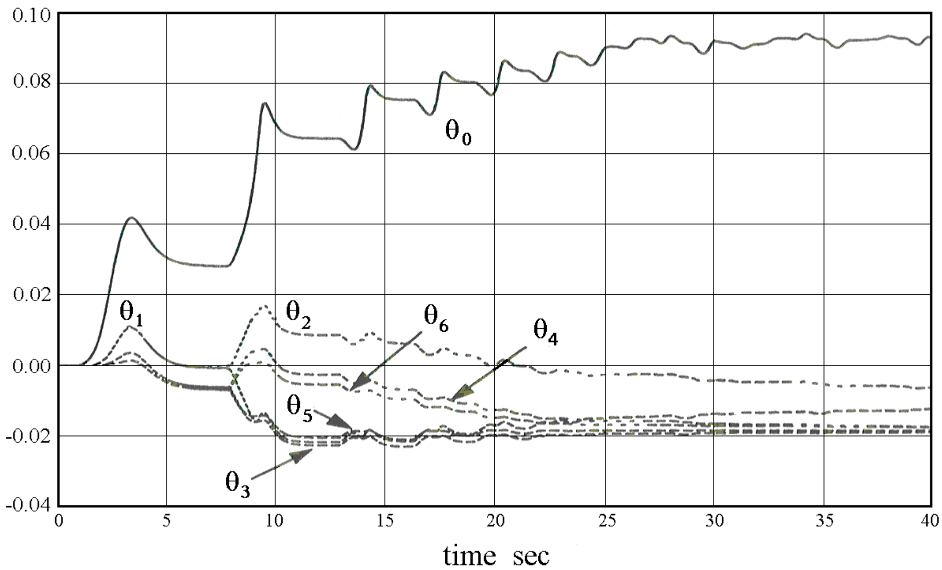

Figure 3- On-line estimation of the identified hysteretic system parameters, obtained through swept-sine-input excitation

In "Figure (4)", the on-line prediction of $\hat{Q}(s)$ (dashed line) is compared to the actual value of $Q(x(s), \dot{x}(s))$ (solid line). The phase plots of $z(s)$ (solid line) and $\hat{z}(s)$ (dashed line) versus $x(t)$ (horizontal axis) are shown in "Fig. (5)". The agreement between the measured and identified hysteretic restoring force is extremely good over the whole domain of the motion as observed from "Fig. (4)" and "Fig. (5)". That is, the algorithm is capable of accurately replicating actual system motion immediately after the start of tracking. Only a few time samples are required for the accurate tracking of the observed response. Even when the identified parameters had not settled to their final value, their instantaneous combinations were able to generate extremely good agreement within that range of local excitation.

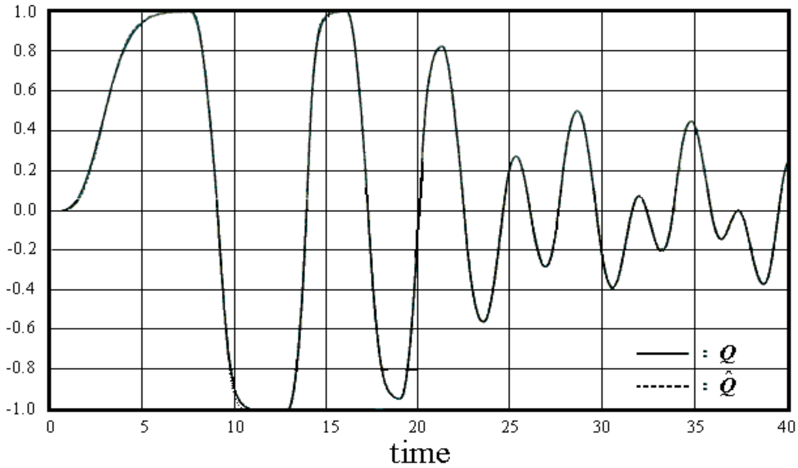

Figure 4- Comparison between the measured, $Q(t)$, and identified, $\hat{Q}(t)$, restoring force under a swept-sine wave excitation

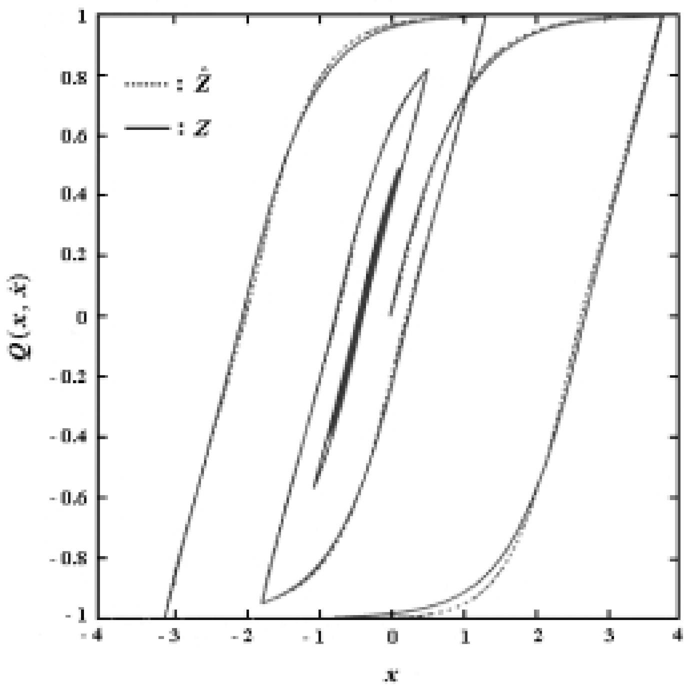

Figure 5- Phase-plane comparison of the measured, $Q(t)$, and identified $\hat{Q}(t)$ restoring force under a swept-sine wave excitation

4.4. Model Validation Procedure

The model identified in section 4.3 should be validated. The parameters θ_i , i = 0....6 are frozen to the values obtained during the identification phase. The system is subjected to a zero-mean-wide-band random excitation $u(t)$ of rms level σ_0, "Fig. (6)". Using the system response data and the parameter values obtained under swept-sine excitation, the predicted values of $\hat{Q}(s)$ are computed. "Figure (7)" shows a very good agreement between the actual $Q(x, \dot{x})$ with the predicted $\hat{Q}$ in spite of the adaptation algorithm being turned off. The phase plane plots of z (solid line) and $\hat{z}$ (dashed line) versus x (horizontal axis) is shown in "Fig. (8)". Again, the agreement between actual and predicted values is very good. Similar results are obtained when the level of excitation is $\sigma \ll \sigma_0$, σ_0 being the random excitation level used in previous validation test.

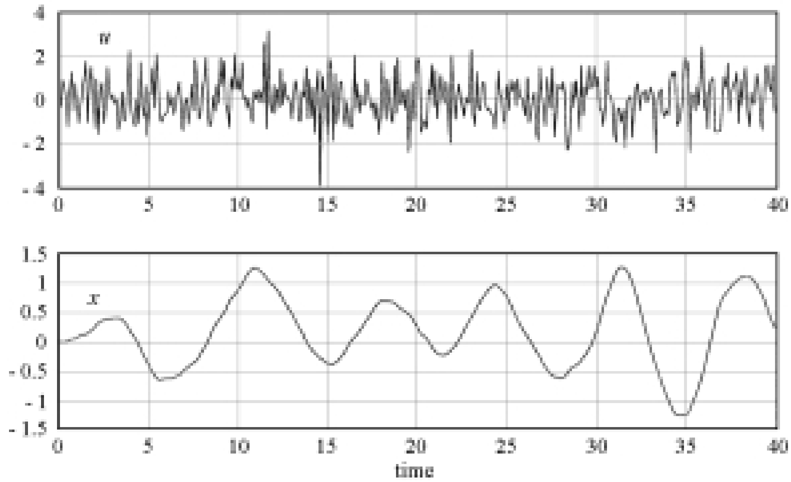

Figure 6- Time-record of system response under wide band random excitation of level σ_0

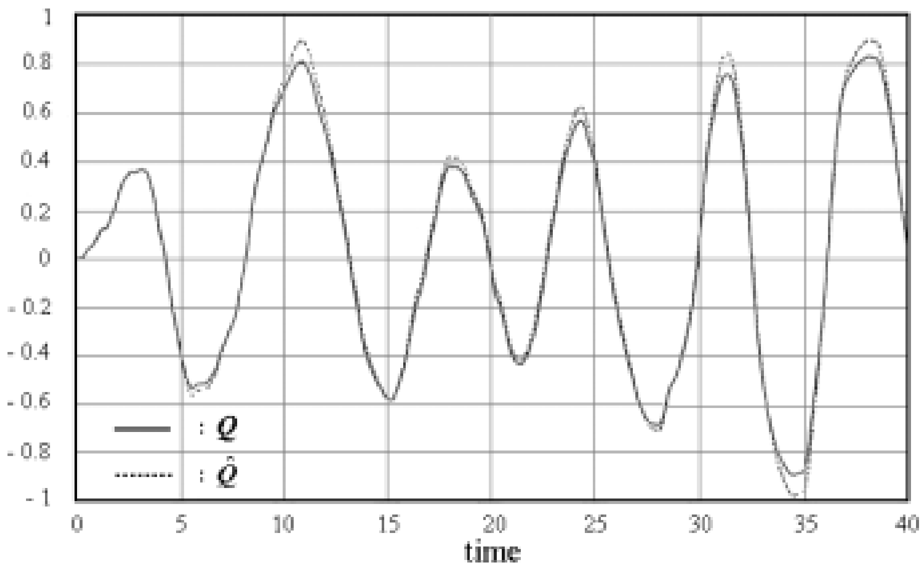

Figure 7- Time-record of the measured and predicted hysteretic restoring force under wide band random excitation of level σ_0 and adaptive scheme turned off

"Figure (3)" shows that the parameters reach a steady state after 20 ~25 seconds. This corresponds to actual CPU time of 0.7 ~ 0.8 seconds, on a 60 MHz machine. This estimate includes the time required to store the data in the ϕ vector. In turn, the actual CPU time can be reduced significantly on a faster machine. "Figure (4)" shows that tracking and prediction of the restoring force is performed much faster: The output tracking is achieved within few cycles of iterations, corresponding to CPU of few milliseconds on the same 60 MHz machine.

Therefore, as an active control, the on-line prediction of the unknown restoring force is achieved a lot faster than parameter convergence.

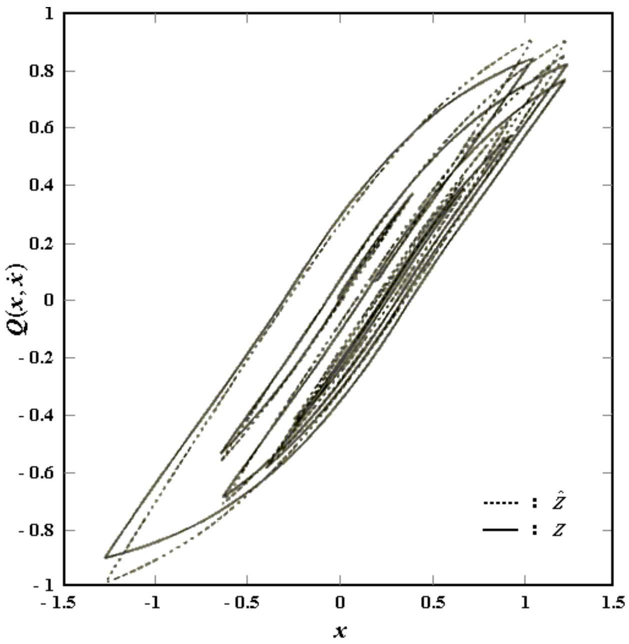

Figure 8- Phase plane plots of the measured and predicted hysteretic restoring force under wide band random excitation of level σ_0 and adaptive scheme turned off

5. DISCUSSION

In all simulations, the estimation scheme is very successful. Under the swept-sine input wave, the model parameters converged within about six cycles. The convergence can be considerably accelerated by introducing a persistent input excitation; for example, a wide-band random excitation is ideal since it will fully excite the system to be identified.

In the present adaptive scheme, the variables affecting the learning rate and the parameter bounds are chosen by trial and error, along with a priori knowledge of the approximate bound on the norm of the system parameter vectors θ^*. In this particular adaptive scheme, the projection bound was not necessary in these simulations. Setting the learning rate too high causes serious algorithm instability with this projection method because the parameters are continuously pushed far outside the bounded parameter space and projected back to the bound at each time step. Other adaptive schemes may contain adaptive learning rates and prove to be more successful.

6. CONCLUSIONS

A method is presented for the on-line identification of hysteretic systems under arbitrary dynamic environment. It is shown through the use of simulation studies that the proposed approach can yield reliable estimates of the hysteretic restoring force under a very wide range of excitation levels and response ranges. The method is ideally suited for on-line control applications involving time-varying non-linear systems typically encountered in the applied mechanics field.

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